

to time decreases during the lapping operation. This feature is of particular importance in gas-bearing performance; typical operating clearances in a high-performance gas bearing are 40 $\mu\text{in.}$, and these clearances are considerably reduced under load. If at any time the bearing were to be overloaded and moving parts touched, wear particles could be generated and cause catastrophic failure of the gas bearing. A bearing made of boron carbide with dry-lapped running surfaces, however, reduces this possibility considerably.

Conclusions

The feasibility of the SLAM gyro is very dependent upon the magnitude and stability of drift coefficients D_F , D_U , D_K , and D_R of Eq. (1). Due to the relationship stated in Eq. (6), to achieve this requisite with a low angular momentum means that each term's contributions to uncertainty torque about the output axis must be small compared to the signal torque. This must be true in a benign as well as an extremely dynamic environment (external accelerations and angular accelerations). To quantify small, an input rate of 1 meru to a gyro with an angular momentum of 8000 dyne-cm-sec results in a torque about the output axis of about 600 $\mu\text{dyne-cm.}$ The achievement of a torque uncertainty about the output axis due to D_F , D_U , D_K , and D_R of this value is the technology of the SLAM gyro.

The SLAM gyro is well suited for strapdown guidance system applications for the following reasons: a) the ability of the angular momentum generator to withstand high angular rates; b) a low angular momentum, which means the TG generated torque to balance the torque due to the input rates is low when compared to a unit with a high angular momentum; c) because of item b and the use of a dual permanent-magnet torque generator, the power required by the torque generator for high rates is extremely low (less than $\frac{1}{4}$ w for 60°/sec); d) the permanent-magnet torque generator design has the linearity, hysteresis, and scale factor stability commensurate with the requirements of high accuracy strapdown systems.

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Frequency Stability Characteristics and Stabilization Techniques in an SSB-FDMA/PhM Multiple Access System

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The system is described and its frequency stability characteristics and stabilization techniques are investigated and analyzed. Utilizing Fourier analysis techniques, the causes of the test tone frequency instability are identified and measured. The system's present AFC unit is described in detail and its shortcomings from the standpoint of reducing the instability factors are presented. Alternate AFC configurations are discussed and the most promising technique (called the reference carrier technique) found to date is described in detail. This entails a description of the instrumentation of the technique and its advantages and disadvantages from a system standpoint. Also the equations that determine the degree of stability compensation are derived and the requirements for optimum compensation are determined. Measurements are presented to show the degree of compensation obtained and the various factors that affect it. It was found that the technique has an amplification effect on thermal noise thus causing an increase in the noise components at frequencies greater than the test tone frequency. Because of the aforementioned effect and the fact that the baseband noise spectrum varies linearly with frequency, the limiting condition for the reduction of the instability is thermal noise.

Introduction

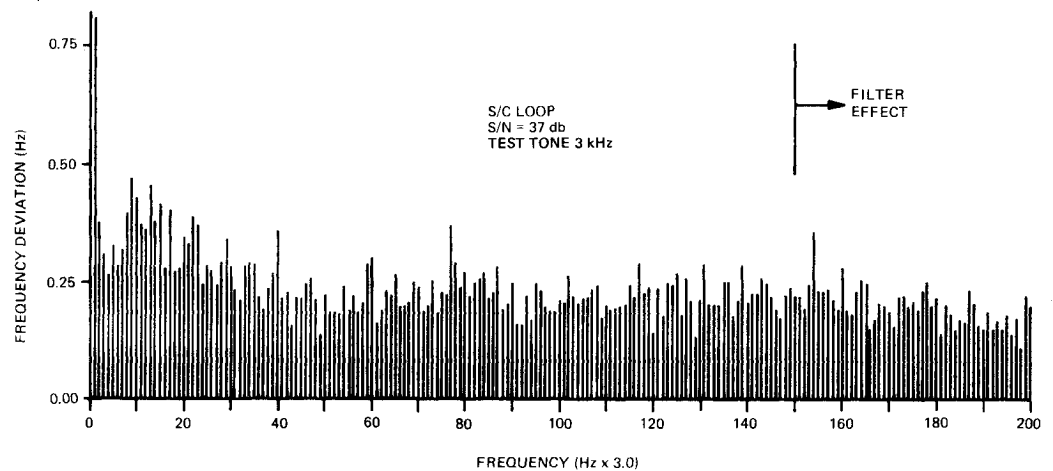
ONE of the communication experiments that is being conducted on a series of Applications Technology Satellites (ATS) utilizing three ground stations (Mojave, California; Rosman, North Carolina and Cooby Creek, Australia) is the testing of a unique SSB-FDMA/PhM multiple access system.

Presented as Paper 70-411 at the AIAA 3rd Communications Satellite Systems Conference, Los Angeles, Calif., June 6, 1970; submitted August 17, 1970. Many of the concepts and test data presented in this paper were obtained from individuals at ATS headquarters located in GSFC and personnel from the three ground stations; Mojave, California, Cooby Creek, Australia and Rosman, North Carolina. In particular I would like to thank E. Metzger, Operations Chief and R. Darcey, Chief of the ATS program for their cooperation and aid in obtaining the required test data.

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The experiment is being conducted on two spin stabilized geostationary satellites (ATS-1 and ATS-3) as a part of the NASA/Goddard Space Flight Center Applications Technology Satellite Program. During the test phase, it was noted that an excessive amount of short-term frequency variations were present on the test tone at the output of a receive multiplex channel. These variations are due in part to incidental angle modulation impressed on the test tone

Fig. 2 Normal system configuration, error correction disabled.



could be determined. Of the two components measured, the 60-Hz (and multiples) components were at least 25 db stronger than the harmonic components. Also, strip chart recordings of the received test tone level indicated the presence of strong 1.6-Hz components and multiples thereof. The 1.6-Hz value is the spin frequency of the spin stabilized satellites.

Because the resolution and bandwidth limitations of the above measurement techniques, another method utilizing Fourier techniques was developed¹ to obtain a more precise quantitative measure of frequency stability. A test tone error spectrum is determined by calculating the amplitude spectrum of a FM discriminator output signal. The amplitude spectrum is a graph of frequency versus the magnitude of the Fourier coefficients of the frequency components existing in the complex signal emanating from a discriminator that is tuned to the test tone frequency. Hence, this frequency produces a direct current term at the output of the discriminator.

In addition, a histogram of the frequency of occurrence vs frequency is also computed and displayed. The histogram represents the frequency distribution of the input signal to the discriminator over an interval of time equivalent to the sample period. The spread of the undesired frequency modulation of the submultiplexed tone is depicted by the histogram. Hence, the spectrum essentially shows the various factors that cause the instabilities and the histogram gives a quantitative measure of the frequency instability. It follows that the standard deviation (σ) of the histogram is a good measure of the frequency instability of the tone.

An example of the resulting test tone error spectrum is shown in Fig. 2. From this type of spectrum and from other methods, it was determined that the following factors affect the test tone frequency instability: (1) 60-Hz sidebands and multiples thereof (incidental modulation); (2) spin modulation components (approx. 1.6 Hz) due to S/C spin effect; (3) basic oscillator $1/f$ phase noise; (4) quadrature components of the normally distributed thermal noise whose magnitude is determined by the ground receiver system noise temperature; (5) frequency offsets of the system oscillators; (6) Doppler shifts.

Because of the extremely low bandwidth of the AFC loop, it follows that only very low frequency rates can be handled by the loop. It has been found that items (5) and (6) fall into the low rate category; hence, the AFC loop can negate their effect. However, the effect of the other four factors cannot be negated so they cause short term variations of the test tone frequency.

The 60-Hz and multiple sidebands are inserted in the system at the SSB ground transmitter. These signals are impressed on the frequency standard by incidental FM modulation. They become a problem because the resulting modulation index is effectively multiplied by a factor of over 800 in

the process of translating the baseband to the 6-GHz frequency range.

The 1.6-Hz spin modulation sidebands are impressed on the 6-GHz signal because of either an electrical or mechanical phase misalignment¹ of the S/C receive antenna with respect to its geometric phase center. As a result, as the S/C spins, there will be a change in the relative phase of the received 6-GHz signal which will eventually be translated to an angle modulation of the received test tone.

The effect of the error correction loop on the output amplitude of the 60-Hz and 1.6-Hz components can be determined by computing the sinusoidal transfer function for a total time lag of about 0.27 sec. For this value of time the sinusoid will peak at a frequency of 1.85 Hz and multiples thereof. In the 60-Hz region the sinusoid will peak at 61.05 Hz. From the aforementioned values, it follows that the error correction unit does, in fact, amplify the 1.6-Hz and 60-Hz sidebands by almost a factor of 6 db.

The $1/f$ phase noise is produced in the various oscillators of the units making up the over-all system. This type of noise is characteristic of all oscillators. Ideally, an oscillator should produce a single line spectrum. In actual practice the continuous $1/f$ spectrum is obtained. Since the error correction unit has a sinusoidal characteristic, it follows that the $1/f$ spectrum would be drastically reduced near the test tone frequency.

Thermal (idle) noise is present in all systems. It can cause short-term frequency variation of the test tone by the vector addition of the signal vector and the quadrature component of the noise vector. Tests were performed to determine the amount of frequency instability caused by the quadrature component. This was accomplished by determining the σ of the frequency deviation histogram as a function of the multiplex channel S/N value. The values of interest lie in a range of 30 db to 40 db. The σ values for 30 db, 34 db, and 40 db are 4.01 Hz, 2.51 Hz, and 1.24 Hz, respectively. The above σ values set a lower limit on the test tone short-term frequency stability.

The long-term frequency variations are mainly due to Doppler shifts and frequency offsets of the system oscillators. The former factor for the case of the geostationary satellites will be less than 100 Hz. The latter factor is mainly caused by the frequency offset (multiplied by 380) of the S/C master oscillator.

Possible Solutions

The major limitation of the present AFC system is the bandwidth constraint due to the large transport lag. This limitation would be present in any feedback loop that must operate over a S/C path for a geostationary satellite. Hence, alternative AFC systems would only compensate for long-term frequency drifts but would not affect the short-term frequency instabilities.

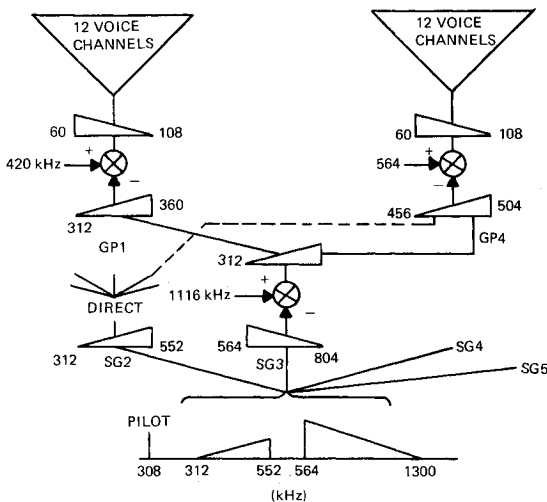


Fig. 3 Multiplex system and carrier insertion points.

Various types of AFC systems were analyzed² on the basis of their steady-state error response to a frequency ramp of unit slope. The types considered were type zero, type zero with error correction (present system), type one and type two units. From the analysis it was concluded that the type one system could replace the present system. The main advantage in this change is that the error correction loop can be eliminated. This loop has a sinusoidal transfer function which causes a gain of almost 6 db at the spin modulation and 60-Hz sidebands.

Because of the bandwidth restrictions, the above systems would not materially affect the short-term frequency instability which is the factor that limits system performance. Also, none of the systems compensate or reduce the effects of the large transport lag. Therefore, the systems were not instrumented because their complexity-to-improvement trade-off value was considered too low. Another alternative that was vigorously pursued was to improve the spectral purity of the frequency standard in the SSB transmitter. This approach reduced but did not eliminate the effects of the 60-Hz sidebands. This alternative was more a remedy rather than a different approach. A reference carrier technique was eventually developed³ that could correct some of the factors that cause short-term frequency instability, regardless of propagation time.

Reference Carrier Technique

This technique is based on the instantaneous compensation of the received FDM baseband. The point at which the frequency stability is really pertinent is at the demodulation point of the receive FDM unit. Therefore, it follows that if the received FDM baseband could be demodulated with respect to a reference carrier that contained aberrations identical to those imposed on the FDM information signals, the resultant frequency distortion values would be reduced.

The method may be implemented by transmitting a carrier with a specific number of channels. At the receiver this carrier is injected into the group or supergroup sections of the receive multiplex unit. This method would apply to the 240-channel multiplex unit at the Mojave and Cooby Creek stations. A functional representation of the multiplex unit is shown in Fig. 3. In operating with supergroup 2, the group carrier (564 kHz) is employed as the reference (specific carrier for demodulating a given number of multiplex channels). For operation with supergroups 3-5, these respective carriers must be employed. It is not permissible to employ the group reference carrier when operating with supergroups 3-5. In this case, the frequency error at the output of the mixer will

double because of the relative phase of the input frequency errors.

The major advantages of the carrier technique is that it can compensate for frequency variations regardless of the magnitude of the S/C transport lag and the magnitude of the incidental modulation present on the signal. The amount of rejection only depends on the difference in the modulation parameters of the FDM signal and carrier and on their relative time displacement τ . The major disadvantage is that it requires altering the standard receive multiplex units and it increases the operational complexity between participating stations. This complexity involves the choice of the correct carrier to transmit which is a function of the number and baseband location of the channel group.

An alternate instrumentation procedure could be employed for the reference carrier technique. This would involve extracting the frequency aberrations from one of the pilot tones and inserting them on a locally generated reference carrier. This would involve a frequency synthesizer, mixers, and selective amplifiers.

Mathematical Description of Reference Carrier Technique

As already stated, the technique consists of utilizing a carrier to mix down to a required frequency that has been subjected to the same perturbations as the desired signal. Let the desired signal be noted as

$$A_1 \cos[\omega_1(t) + \theta_1(t)] \quad (1)$$

and the reference carrier as

$$A_2 \cos[\omega_2(t - \tau) + \theta_2(t - \tau)] \quad (2)$$

where $\theta_1(t)$ is the angular modulation applied to ω_1 and $\theta_2(t)$ is the angular modulation applied to ω_2 . For the general case assume that $\theta_1(t) \neq \theta_2(t)$, τ is the relative time displacement between the two waves.

If the two signals Eqs. (1) and (2) are applied to a mixer and the difference term extracted, it will be

$$(A_1 A_2 / 2) \cos[(\omega_2 - \omega_1)t - \omega_2 \tau + \theta_2(t - \tau) - \theta_1(t)] \quad (3)$$

It is noted that the difference term will consist of the difference frequency $\omega_2 - \omega_1$ a phase shift $\omega_2 \tau$ and a difference term between the angle modulation functions. Let

$$\theta_2(t - \tau) = M_2 \sin \omega_m(t - \tau) \quad (4)$$

$$\theta_1(t) = M_1 \sin \omega_m t \quad (5)$$

M_2 and M_1 are the modulation indices applied to ω_2 and ω_1 , respectively. The resultant angle modulation term will be

$$M_2 \sin \omega_m(t - \tau) + M_1 \sin(\omega_m t + \pi) \quad (6)$$

The terms of Eq. (6) are two vectors whose magnitudes are M_2 and M_1 and whose arguments are $\omega_m(t - \tau)$ and $(\omega_m t + \pi)$, respectively. The magnitude of the resultant vector C is

$$(M_2^2 + M_1^2 - 2M_2 M_1 \cos \omega_m \tau)^{1/2} \quad (7)$$

and its argument is

$$\tan^{-1} \left[\frac{M_2 \sin \omega_m(t - \tau) - M_1 \sin \omega_m t}{M_2 \cos \omega_m(t - \tau) - M_1 \cos \omega_m t} \right] \quad (8)$$

If $\tau = 0$, the over-all expression reduces to

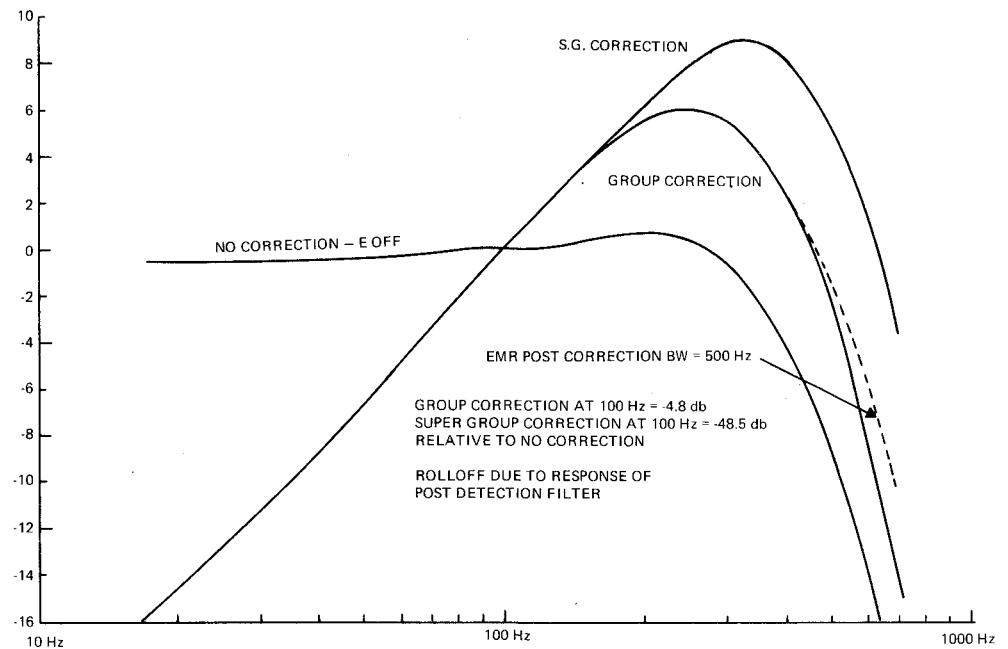
$$(M_2 - M_1) \sin \omega_m t \quad (9)$$

If τ is finite but $M_2 = M_1$, it reduces to

$$[2M_2 \sin(\omega_m \tau / 2)] \cdot \cos(\omega_m t - \omega_m \tau / 2) \quad (10)$$

It follows that perfect cancellation occurs only if $\tau = 0$ and $M_2 = M_1$. Equation (10) closely approximates the actual condition for the operational system. In this case it is expected that the angle modulation sidebands will increase

Fig. 4 Carrier correction response.



sinusoidally for increasing values of f_m . The M factor will be attenuated for values of f_m less than $1/6\tau$. For higher values there will be a gain in M reaching a maximum of $2M$ at an f_m of $1/2\tau$.

Test Results

To determine the effect of the reference carrier technique on thermal noise, incidental modulation and the $1/f$ phase noise factors, tests were performed in various loops and test configurations. The effect of thermal noise was determined by instrumenting a loop consisting only of the transmit and receive multiplex units. At the transmit unit, thermal noise was injected into the system so that the exact S/N (test tone to thermal noise ratio) obtained in the S/C loop was realized.

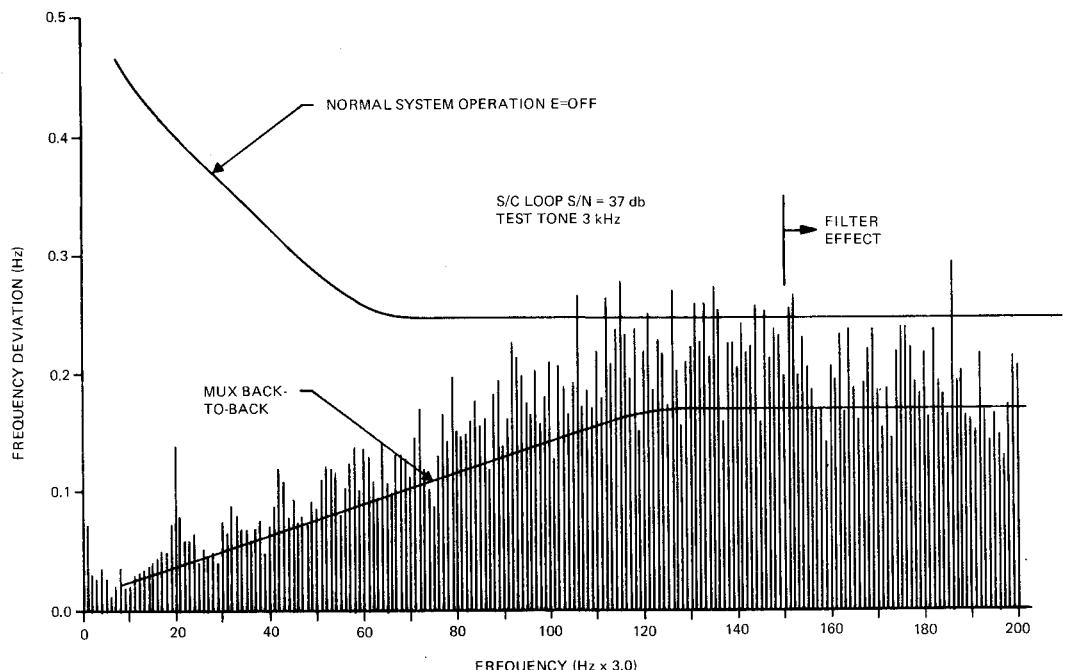
The response of the technique to the incidental modulation components was determined by impressing an FM signal of varying modulation frequencies on the test tone and reference

carrier. The resulting response is shown in Fig. 4. Utilizing the response level at 100 Hz for the uncorrected condition as a reference, it was determined that the response for the group correction was -4.8 db and for the supergroup correction -48.5 db. The sinusoidal variation of attenuation with modulation frequency is evident as predicted.

The 43.7-db difference in the attenuation factors is mainly due to the values of the τ factors for the two correction methods. A narrow band filter is utilized with group correction. The relative time delay caused by this filter is much greater than the value obtained for the wide band filter employed for the supergroup correction.

A characteristic error spectrum is shown in Fig. 2. The spectrum is basically flat from 120 Hz to about 450 Hz. At higher frequencies the filters cause a definite roll-off. At lower frequencies the spectrum tends to increase due to the spin modulation components $1/f$ phase noise and components being emitted by the SBB transmitter. Ideally, the spectrum should be triangular since the noise voltage at the output

Fig. 5 S/C loop with supergroup correction — error correction disabled.



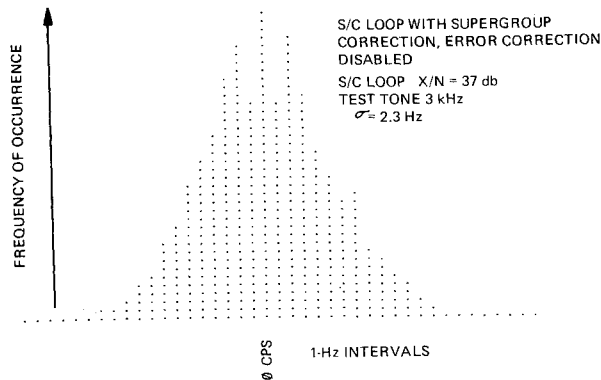


Fig. 6 Frequency deviation histogram.

of an FM discriminator varies as kf (k is a constant and f is the baseband frequency). This triangular shape is realized in the multiplex back-to-back loop where only thermal noise is present.

Figure 5 is a representative plot of the error spectrum for supergroup correction of the normal condition shown in Fig. 2. Figure 6 is the corresponding histogram of the frequency distribution. Also shown in Fig. 5 are the lines that define the peak components of the error spectrums for the normal S/C loop and multiplex back-to-back loop configurations. From Fig. 5 it is seen that the reference carrier technique eliminates all of the noise components in the lower part of the spectrum but has an amplification effect on the thermal noise. This latter effect was checked by performing a test in the multiplex loop at an S/N of 37 db and 43 db with and without the reference carrier technique. In both S/N cases, the σ factors when using the technique were higher.

Table 1 lists the results of tests conducted while employing a 3-kHz test tone frequency various S/N values and different loop configurations. The resulting σ 's for normal and carrier reference conditions (supergroup and group corrections) are also listed. It is noticed that the values for the correction conditions are almost the same for the multiplex and S/C loop configurations. Hence, the thermal noise factor limits the final value of σ when employing the reference carrier technique in the S/C loop configuration. An increase of S/N of 6 db causes a reduction of 6.2 db in the σ factor (2.31 Hz to 1.12 Hz).

The limitation in the improvement of the σ factor when employing the carrier correction technique is mainly due to two factors: 1) the apparent amplification of the thermal noise component by the technique; and 2) the triangular shape of the baseband noise voltage. Both of these factors tend to increase the value of σ . This increase can be seen from the expression employed to compute σ

$$\sigma^2 = \sum_{j=1}^n (x_j - \bar{x})^2/n \quad (11)$$

where x_j are the sample values, \bar{x} is the mean of the sample values, and n is the number of sample points.

From Eq. (11), it is noted that σ is a direct measure of the distance that the sample values are located from the mean. It follows that an x_j near the mean is not critical as one that is a large distance from the mean. Hence, σ is heavily weighted by values at relatively large distances from the mean. Elimination of noise components in a given frequency interval near the mean is not as effective in reducing σ as elimination of the components in the same interval at a distance of 500 Hz from the mean. Since the baseband noise varies as kf , the increase in noise at the high end of the baseband region limits the amount that σ can be reduced. A direct reduction of this noise can only be realized by an increase in the S/N value.

Table 1 Reference carrier test data

Test loop	MUX S/N, db	Normal system config., σ -Hz	Carrier Reference		Test run
			Super group σ , Hz	Group σ , Hz	
MUX BB	37	1.78	1
S/C	37	3.98	2.52	3.57	1
MUX BB	37	1.66	2.36	...	2
S/C	37	3.16	2.31	...	2
MUX BB	43	0.91	1.23	...	2
S/C	43	3.0	1.12	...	2
S/C	37	3.74 ^a

^a With error correction. All other cases, error correction is off.

Summary

The over-all system AFC unit consists of a primary AFC loop and an open loop system defined as an error correction loop as shown in Fig. 1. The latter loop has a sinusoidal response that is a function of the S/C transport lag. For the 0.27-sec lag value obtained, it can be shown that the error correction loop amplifies the spin modulation and the 60-Hz components. It also attenuates the $1/f$ noise. From various tests, it has been shown that the over-all AFC unit can compensate for long term frequency drifts (S/C oscillator drifts or Doppler).

A type 1 AFC unit could replace the present unit. Both systems produce a finite steady-state error for an input frequency ramp of unit slope. The type 1 unit can eliminate the unwanted amplification of the 1.6-Hz and 60-Hz components. However, neither can eliminate or substantially reduce the factors that cause the short-term frequency instability.

A reference carrier technique was developed that could reduce the previous instability regardless of propagation time. The major advantage of this technique is that the degree of reduction only depends on the difference in the modulation parameters on the FDM signal and carrier and on their relative time displacement τ . Its disadvantages are as follows: 1) it must generate and transmit an additional carrier or carriers; 2) it requires an additional amount of transmitter power; 3) it increases system complexity by requiring a frequency synthesizer at the transmitter and an alteration of the standard receive multiplex unit; 4) it decreases the versatility of assigning multiplex channels for the participating stations; 5) it can cause a 6-db increase in the incidental modulation if the value of τ is high enough; and 6) it causes an apparent amplification of the thermal noise in the baseband region.

The standard deviation (σ) of the frequency deviation histogram was chosen as the logical measure of the frequency instability. This factor weighs the noise components in the upper baseband region quite heavily. Because of the amplification effect and the fact that the baseband noise spectrum varies as kf , the limiting condition for the reduction of σ is the thermal noise component. Its reduction can be accomplished by an increase in the test tone to thermal noise ratio.

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